

Governments and Producers in Multinational Markets: A Mixed Oligopoly

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Abstract

This paper extends the existing mixed oligopoly analysis by studying the case where governments do not have the same attitude toward producers' surplus in a multinational market. It demonstrates that governments' emphasis on profitability might discourage them from joining a coordinated privatization process with other governments. Thus the governments do not always face a prisoners' dilemma in privatizing their national firms as suggested previously. It also demonstrates that private sector increases its production when governments appreciate producers' surplus more than consumers'.

Keywords: mixed oligopoly, producers' surplus, multinational markets

JEL Classification: L13, L32

1. Introduction

This paper is inspired in part by the international commercial aviation industry, where governments interact with competing domestic and foreign airlines through national flag carriers. While some airlines are public enterprises, a growing number are private firms (Doganis 2001) maximizing their profits and not their respective societies' social welfare. This trend has also caused many governments to rethink their approach to international aviation and to emphasize profitability. The present article investigates how governments' attitudes toward profitability and producers' surpluses would influence market and firms' output-setting decisions. To do so, it utilizes a mixed oligopoly market structure.

Introduced by Merrill and Schneider (1966) a mixed oligopoly market includes both public firms and private ones. This analysis became particularly popular after the collapse of the Soviet Union in late 1980's (DeFraja and Delbono 1989 and 1990). Since then, a growing number of authors have utilized it as a framework to

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study domestic markets with both public and private firms in different settings (Harris and Wiens 1980, Sertel 1988, Cremer, Marchand and Thisse 1989, 1991, Fershtman 1990; Poyago-Theotoky 2001, Nilssen and Sørgard 2002, Myles 2002). The increasing popularity of mixed oligopoly has inspired several researchers to use it in analyzing domestic markets with foreign competition (Fjell and Pal 1996, Pal and White 1998, Fjell and Heywood 2002, Sepahvand 2002). Most recently these applications have been extended to include international markets, where public and private firms of different nationalities compete and governments interact with each other using their public firms as strategic trade instruments (Pal and White 2003, Dadpay and Heywood 2006, Fujiwara 2006, Sepahvand and Cornes 2007).

In these studies, authors assume that the governments have a similar approach toward profitability and consumers' surplus, from which they do not deviate. These studies presume that the goal of governments is to maximize their respective social welfare function, which is the sum of consumers' surplus and producers' surplus (DeFraja and Delbono 1989). In reality, governments are not homogenous in their treatment of producers' surplus. Some attach a higher (or lower) significance to producers' surplus than others. Nowhere else this discrepancy is more vividly apparent than in commercial aviation. For example in Persian Gulf region governments such as UAE and Qatar, emphasize the profitability of their national airlines while others such as Iran focus on the affordability of international travel for their citizens.

The mixed oligopoly analysis has been particularly implemented to study the consequences of privatization in domestic markets (DeFraja and Delbono 1989, Fjell and Heywood 2002) as well as international ones. Pal and White (2003) and Dadpay and Heywood (2006) argue that in a multinational mixed oligopoly governments face a prisoners' dilemma in liberalizing the industry since privatizing one country's public firm reduces its social welfare but increases its rival's. Dadpay and Heywood (2006) demonstrate that public firms' reactions in such market are symmetric and private firms act similarly and independently from their nationality. The present article shows that these results hold only when governments treat producers' surplus correspondingly. Governments' differences regarding profitability alter the symmetry of equilibrium results; influence private firms' conduct and affect governments' decision to privatize their public firms unilaterally.

While motivated by international commercial aviation industry this article also is inspired by the void in the existing mixed oligopoly literature regarding the governments' attitudes toward producers' surplus. It offers a multinational model to study governments' conduct where producers' surplus is not as significant for all governments present in the market. It outlines how governments' differentiated attitudes change the equilibrium in such a market and alter the consequences of privatization. It finds out that the previous results are valid only when governments have the same attitude toward producers' surplus. When the importance of producers' surplus varies across countries then the consequences of either unilateral or coordi-

nated privatization are ambiguous and a function of market structure. In some cases governments are able to avoid prisoners' dilemma however this does not mean that governments universally would welcome a coordinated liberalization.

The remainder of this paper is organized as follows. Section 2 sets up the model and presents the equilibrium results. Section 3 presents a series of propositions that capture the unique aspects of our equilibrium compared to those in already existing models. Section 4 uses the model to return to the policy issues such as open door policy and privatization that interested previous researchers; Section 5 discusses the robustness of the model and potential ways for improvement. The last section summarizes the discussion and concludes the article.

2. The Model

Consider a multinational market in which customers from two nations purchase a homogenous commodity produced by competing private firms and public ones. There are one domestic public firm, one foreign public firm, m domestic private firms and n foreign private firms in this market interacting with each other and customers. Governments influence the market by determining their respective public firms' outputs. All firms have identical quadratic cost functions:¹

$$(1) \quad C(q) = (1/2)k(q)^2 + f$$

There is a fixed cost f and a linear marginal cost kq , where $k > 0$ is a constant. This specification of costs represents increasing marginal costs.

Let q_o^d be the output of the public domestic firm, q_o^f be the output of the foreign public firm, q_i^d be the output of a private domestic firm ($i = 1, \dots, m$) and q_j^f be the output of a foreign private firm ($j = 1, \dots, n$). Domestic and foreign private firms profit functions are:

$$(2) \quad \pi_i^d = Pq_i^d - (1/2)k(q_i^d)^2 - f$$

$$(3) \quad \pi_j^f = Pq_j^f - (1/2)k(q_j^f)^2 - f$$

And domestic and foreign public firms profit functions are:

$$(4) \quad \pi_o^d = Pq_o^d - (1/2)k(q_o^d)^2 - f$$

$$(5) \quad \pi_o^f = Pq_o^f - (1/2)k(q_o^f)^2 - f$$

Assuming a linear inverse demand curve:

¹ Quadratic cost functions are often utilized to study commercial aviation industry, see Borenstein and Rose (1994).

$$(6) \quad P = a - \left(q_o^d + q_i^d + \sum_{i=1}^m q_i^f + \sum_{j=1}^n q_j^f \right)$$

As a consequence, total consumer surplus from the industry is:

$$(7) \quad CS = (1/2) \left(q_o^d + q_o^f + \sum_{i=1}^m q_i^d + \sum_{j=1}^n q_j^f \right)^2$$

Consumers from foreign and domestic countries share the consumer surplus in the market. Thus countries' social welfare functions would be written as:

$$(8) \quad W_o^d = \theta(1/2) \left(q_o^d + q_o^f + \sum_{i=1}^m q_i^d + \sum_{j=1}^n q_j^f \right)^2 + \gamma_d \left(\pi_o^d + \sum_{i=1}^m \pi_i^d \right)$$

$$(9) \quad W_o^f = (1 - \theta)(1/2) \left(q_o^d + q_o^f + \sum_{i=1}^m q_i^d + \sum_{j=1}^n q_j^f \right)^2 + \gamma_f \left(\pi_o^f + \sum_{j=1}^n \pi_j^f \right)$$

Where θ is the share of total output consumed by customers from the domestic country, γ_d and γ_f represent the relative importance of producers' surplus in each country's social welfare. When governments do not discriminate between producers' surplus and consumers' surplus, $\gamma_i = 1$, for $i = f$ or d , and when any of the governments considers producers' surplus to be more important than consumer surplus, $\gamma_i > 1$, for $i = f$ or d .

Social welfare functions do not include the profit earned by firms from the other country or the consumer surplus enjoyed by citizens from the other country. Governments maximize their respective social welfare functions by choosing public firms' outputs accordingly. Global social welfare would be the sum of producers' surplus and consumers' surplus and can be written as:

$$(10) \quad W_o^g = (1/2) \left(q_o^d + q_o^f + \sum_{i=1}^m q_i^d + \sum_{j=1}^n q_j^f \right)^2 + \left(\pi_o^d + \pi_o^f + \sum_{i=1}^m \pi_i^d + \sum_{j=1}^n \pi_j^f \right)$$

Notice that governments can only discriminate between producers' surplus and consumers' surplus at a national level and their attitudes are not enforced globally.

Following the Cournot-Nash assumption, firms set their outputs simultaneously. Domestic and foreign private firms maximize their profits, while domestic and foreign public firms choose their outputs to maximize their respective country's social welfare. Jointly solving the series of first order conditions yields the following equilibrium values for firms' outputs, equilibrium price and equilibrium total output.

A domestic private firm’s output equals that of a foreign private firm:

$$(11) \quad q_i^{d*} = q_j^{f*} = \frac{a[\gamma_d \gamma_f (k + 1) - (1 - \theta)\gamma_d - \theta\gamma_f]}{\gamma_d \gamma_f [(k + 1)^2 + k(m + n + 2)] - (k + 1)[\gamma_d(1 - \theta) + \gamma_f\theta]}$$

It must be noted that second order conditions are:

$$\frac{\partial^2 \pi_i^d}{\partial q_i^{d^2}} = -(k + 2m) < 0$$

$$\frac{\partial^2 \pi_j^f}{\partial q_j^{f^2}} = -(k + 2n) < 0$$

These conditions always hold since k or the slope of marginal cost is positive as well as m & n , the number of domestic and foreign private firms in the market.

This equilibrium output for private firm recovers a similar finding by Dadpay and Heywood (2006), where they suggest that private firms’ outputs are independent from their nationality. However it does deviate from their suggestion that private firms do not consider their respective countries’ market share in determining their output. Equation (11) demonstrates that this is not true when governments do not treat producers’ surplus similarly.

Maximizing domestic social welfare, the domestic public firm’s output is:

$$(12) \quad q_o^{d*} = \frac{a\{(k + 1 - m)[\gamma_d \gamma_f (k + 1) - \gamma_d(1 - \theta)] + \gamma_f\theta[k(m + n + 1) + m + 1]\}}{(k + 1)^2[\gamma_d \gamma_f (k + 3) - \gamma_d(1 - \theta) - \gamma_f\theta] + \gamma_d \gamma_f k(k + 1)(m + n)}$$

And the foreign public firm’s output is:

$$(13) \quad q_o^{f*} = \frac{a\{(k + 1 - n)[\gamma_d \gamma_f (k + 1) - \gamma_f\theta] + \gamma_d(1 - \theta)[k(m + n + 1) + n + 1]\}}{(k + 1)^2[\gamma_d \gamma_f (k + 3) - \gamma_d(1 - \theta) - \gamma_f\theta] + \gamma_d \gamma_f k(k + 1)(m + n)}$$

For domestic and foreign social welfare functions second order conditions are:

$$\frac{\partial^2 W_o^d}{\partial q_o^{d^2}} = \theta - 2\gamma_d - \gamma_d k < 0 \Rightarrow \frac{\theta}{\gamma_d} < 2 + k$$

$$\frac{\partial^2 W_o^f}{\partial q_o^{f^2}} = 1 - \theta - 2\gamma_f - \gamma_f k < 0 \Rightarrow \frac{1 - \theta}{\gamma_f} < 2 + k$$

Since $0 < \theta < 1$ and $k > 0$ these second order conditions hold unless the weights of the producers’ surplus are very close to zero. Since in the real world producers’ surplus is not without significance, then it is realistic to assume that for the purpose of this study these second order conditions always hold and social welfare functions are convex, without having to assume that k is large enough to validate

these conditions. It must be pointed out that although the domestic public firm's output is different from the foreign public firm's output; public firms' outputs are symmetric with respect to market share and the relative significance of producers' surplus.

Summing private and public firm outputs results in the equilibrium total output:

$$(14) \quad Q^* = \frac{a \gamma_d \gamma_f [(m+n+2)k+2]}{(k+1)[\gamma_d \gamma_f (k+3) - \gamma_d(1-\theta) - \gamma_f \theta] + \gamma_d \gamma_f k(m+n)}$$

And the equilibrium price is:

$$(15) \quad P^* = \frac{a(k+1)[\gamma_d \gamma_f (k+1) - (1-\theta)\gamma_d - \theta\gamma_f]}{(k+1)[\gamma_d \gamma_f (k+3) - \gamma_d(1-\theta) - \gamma_f \theta] + \gamma_d \gamma_f k(m+n)}$$

It is apparent that in the case of discriminating between consumers' surplus and producers' surplus firms' actions, private and public, are influenced by how governments treat producers' surplus. It must be noted that domestic and foreign market shares θ or $(1-\theta)$ appear in private firms' outputs in interaction with the relative weight of producers' surplus in domestic and foreign social welfare functions. Thus $\gamma_d = \gamma_f$ recovers Dadpay and Heywood (2006) equilibrium results for private firms' outputs, where these firms are independent of domestic or foreign market share. If domestic customers consume all the quantity available in the market and receive the entire consumer surplus then the foreign government's attitude toward producers' surplus becomes irrelevant. In other word governments' attitude toward profitability influence the market as a function of their population share of consumers' surplus. Appendix A presents the extreme cases for $\theta = 0$ or 1 . The consequences of adopting a differentiated treatment of producers' surplus are not limited to this. The next section outlines how any alterations in either government's attitude toward producers' surplus affect the equilibrium values by causing firms to respond.

3. Market Implications

The model takes into account two exogenous parameters: one is the relative importance of producers' surplus for the governments, and the other is the domestic country's share of consumer surplus or market share. We investigate the influence of both market share and relative significance of producers' surplus on the equilibrium following subsections offer the findings of this analysis.

3.1 The Significance of Producers' Surplus

This paper initiates the idea of incorporating governments' treatment of producers' surplus in a multinational mixed oligopoly. It studies the consequences of any

alteration in governments' attitude and how the equilibrium evolves because of firms responding to such a change. This results in the following proposition:

Proposition 3.1.1. If the relative significance of producers' surplus increases in either country, then both domestic and foreign private firms increase their output at the same rate.

Proof. From equation 11 derivatives of domestic and foreign private firms' outputs with respect to γ_d & γ_f are:

$$(16) \quad \frac{\partial q_d^{d*}}{\partial \gamma_d} = \frac{\partial q_f^{f*}}{\partial \gamma_d} = \frac{a \theta \gamma_f^2 [k(m+n+2) + 2]}{\{(k+1)[\gamma_d \gamma_f (k+3) - \gamma_d(1-\theta) - \gamma_f \theta] + k \gamma_d \gamma_f (m+n)\}^2} > 0$$

$$(17) \quad \frac{\partial q_d^{d*}}{\partial \gamma_f} = \frac{\partial q_f^{f*}}{\partial \gamma_f} = \frac{a(1-\theta)\gamma_d^2 [k(m+n+2) + 2]}{\{(k+1)[\gamma_d \gamma_f (k+3) - \gamma_d(1-\theta) - \gamma_f \theta] + k \gamma_d \gamma_f (m+n)\}^2} > 0$$

These expressions are positive as their denominators are always positive and their numerators constitute only positive elements.

As equations (16) and (17) show, if the significance of producers' surplus increases in the domestic country's social welfare, then both domestic and foreign private firms increase their outputs proportional to domestic share of consumer surplus. That is also the case when producers' surplus becomes more important for the foreign government. The reactions to changes in the domestic government's attitude are symmetric to those caused by changes in the foreign government's attitude. However these are not equal in magnitude.

Proposition 3.1.2. An increase in the relative significance of producers' surplus for the domestic (foreign) government causes the total output of public firms present in the market to decline.

Proof. Total output of public firms present in the market is calculated by adding the domestic public firm's output and the foreign public firm's output, given by equations (12) and (13). The derivative of the public firms' total output is derived with respect to γ_d :

$$(18) \quad \frac{\partial q_o^{d*}}{\partial \gamma_d} + \frac{\partial q_o^{f*}}{\partial \gamma_d} = \frac{-2a\theta\gamma_f^2\{(k+1)^2(m+n+2) + (m+n)[k(m+n+1) + 1]\}}{\{\gamma_d \gamma_f [k^2 + (m+n+4)k + 3] - (k+1)[(1-\theta)\gamma_d + \theta\gamma_f]\}^2}$$

Since the numerator for this expression is negative for all the values of m, n and k and its denominator is always positive then the derivative is always negative.

When producers' surplus significance increases for the domestic government, its public firm reduces its output to increase the equilibrium price. This would increase private firms' profits. Reacting to this increase in equilibrium price the foreign government increases its public firm's output to reduce the equilibrium price

to prevent the loss of consumers' surplus to rival country's firms. However the domestic public firm decreases its output more than the increase in the foreign public firm's output. Thus public firms' total output declines.

Proposition 3.1.3. If producers' surplus becomes more important for any government involved in the market, then equilibrium total output declines.

Proof. The derivatives of total output with respect to γ_d & γ_f are estimated from equation 14. These derivatives are presented in equations (19) and (20):

$$(19) \quad \frac{\partial Q^*}{\partial \gamma_d} = - \frac{a \theta \gamma_f^2 (k + 1) [k(m + n + 2) + 2]}{\{(k + 1) [\gamma_d \gamma_f (k + 3) - \gamma_d (1 - \theta) - \gamma_f \theta] + k \gamma_d \gamma_f (m + n)\}^2} < 0$$

$$(20) \quad \frac{\partial Q^*}{\partial \gamma_f} = - \frac{a(1 - \theta) \gamma_d^2 (k + 1) [k(m + n + 2) + 2]}{\{(k + 1) [\gamma_d \gamma_f (k + 3) - \gamma_d (1 - \theta) - \gamma_f \theta] + k \gamma_d \gamma_f (m + n)\}^2} < 0$$

The numerators of these expressions are negative while their denominators are positive the expressions are negative as a whole.

The change in total output is proportional to the country's share of consumer surplus. The larger a country's share of consumer surplus, the more total output decreases because of an increase in the significance of producers' surplus in that country. According to Proposition 3.1.2, total output of public firms decline if producers' surplus gains prominence in either country. The magnitude of this decline is greater than the sum of individual increases in the private firms' outputs caused by the same event, according to Proposition 3.1.1. This phenomenon is explained by the fact that when private firms increase their outputs, their respective government decreases its public firms' output in order to maximize social welfare by increasing profits through an increased equilibrium price. For this change to occur, the public firm has to offset the total increase in the private firms' outputs by decreasing its output further. Thus equilibrium total output decreases and equilibrium price increases.

The changes in governments' attitudes toward producers' surplus influence the global welfare in ambiguous ways. We derive the derivatives of global welfare with respect to γ_d & γ_f from equation 10 at the equilibrium point. However the resulting expressions are too complicated for identifying their signs. A numerical simulation is conducted to study them. It is apparent that these derivatives are not always positive or negative. For example, when $\gamma_f = 1$ and $\gamma_d > \gamma_f$, then an increase in the relative weight of producers' surplus in the domestic country reduces global welfare if the domestic country's share of consumer surplus is small. Under similar conditions a positive change in the foreign government's attitude toward producers' surplus increases the global welfare. This finding demonstrates that in this model the change in global welfare is the outcome of interactions between several factors, which vary from market structure to market structure.

3.2 Market Share Effects

Studying the influence of market share, Dadpay and Heywood (2006) conclude that the domestic public firm increases its output as the domestic market share increases, but the decline in the foreign public firm’s output at the same time offsets this increase creating a zero net effect. They point out that since the private firms’ outputs are insensitive to any change in the domestic market share, the equilibrium total output does not change at all. As Equation (11) shows, a private firm’s output is a function of the domestic country’s market share. Proposition 3.2.1 summarizes the results.

Proposition 3.2.1. When the domestic (foreign) country’s market share, θ , increases, then a private firm increases its output if, and only if, producers’ surplus is more significant for the domestic (foreign) government than for the foreign (domestic) one.

Proof. The derivatives of a private firm’s output with respect to the domestic market share, θ , and the foreign market share, $1 - \theta$ are derived from equation 11:

$$(21) \quad \frac{\partial q_i^{d*}}{\partial \theta} = \frac{\partial q_j^{f*}}{\partial \theta} = \frac{a \gamma_d \gamma_f (\gamma_d - \gamma_f) [k(m + n + 2) + 2]}{\{\gamma_d \gamma_f [(k + 2)^2 + k(m + n) - 1] + (k + 1)[\theta(\gamma_d - \gamma_f) - \gamma_d]\}^2}$$

$$(22) \quad \frac{\partial q_i^{d*}}{\partial (1 - \theta)} = \frac{\partial q_j^{f*}}{\partial (1 - \theta)} = \frac{a \gamma_d \gamma_f (\gamma_f - \gamma_d) [k(m + n + 2) + 2]}{\{\gamma_d \gamma_f [(k + 2)^2 + k(m + n) - 1] + (k + 1)[\theta(\gamma_d - \gamma_f) - \gamma_d]\}^2}$$

From equation 21, it is apparent that for a private firm to increase its output because of an increase in domestic market share, the condition $\gamma_d > \gamma_f$ must hold. Should foreign market share increase, then for a private firm to increase its output $\gamma_f > \gamma_d$ must be true.

This highlights the role of public firms’ in market. Their reactions are interpreted as government’s efforts to influence market equilibrium in these circumstances. Using equations (12) and (13), we calculate the derivatives of public firms’ outputs with respect to θ . These derivatives for the case of $m = n = 1$ are:

$$(23) \quad \frac{\partial q_o^{d*}}{\partial \theta} = \frac{2 a \gamma_d \gamma_f (2k + 1) [\gamma_d k + \gamma_f (k^2 + 5k + 3) - (k + 1)]}{(k + 1) \{ (k + 1) [\gamma_d \gamma_f (k + 3) - \gamma_d (1 - \theta) - \gamma_f \theta] + 2 \gamma_d \gamma_f k \}^2}$$

$$(24) \quad \frac{\partial q_o^{f*}}{\partial \theta} = - \frac{2 a \gamma_d \gamma_f (2k + 1) [\gamma_d (k^2 + 5k + 3) + \gamma_f k - (k + 1)]}{(k + 1) \{ (k + 1) [\gamma_d \gamma_f (k + 3) - \gamma_d (1 - \theta) - \gamma_f \theta] + 2 \gamma_d \gamma_f k \}^2}$$

It must be noticed that the changes in public firms’ outputs are symmetric with respect to the relative weights of producers’ surplus. Equations (23) and (24) show that the domestic public firm usually increases its output and the foreign public firm usually decreases its output, because of an increase in the domestic market share. Although these changes are usually in the opposite direction and of different magnitudes, their net effect is not equal to zero as suggested in the previous studies.

Proposition 3.2.2. When the domestic market share, θ , increases, then the sum of public firms' outputs increases if, and only if, producers' surplus is more important in the foreign country than in the domestic country.

Proof. Using Equations (12) and (13), the sum of derivatives of public firms' outputs with respect to θ is written as:

$$(25) \quad \frac{\partial q_o^{d*}}{\partial \theta} + \frac{\partial q_o^{f*}}{\partial \theta} = \frac{a \gamma_d \gamma_f (\gamma_f - \gamma_d) \{k^2(m+n+2) + [k(m+n+2) + 2](m+n+1) + 2k\}}{\{(k+1)[\gamma_d \gamma_f (k+1) + \theta(\gamma_d - \gamma_f) - \gamma_d] + \gamma_d \gamma_f k(m+n+2)\}^2}$$

It is apparent that for equation (25) to be positive $\gamma_f > \gamma_d$ must hold.

Please notice when $\gamma_d > \gamma_f$ private firms increase their outputs because of an increase in the domestic market share, θ , while the net effect of the changes in public firms' outputs is negative. This intrigues us to ask in which direction the equilibrium total output would change.

Proposition 3.2.3. When the domestic (foreign) market share, θ , increases, the equilibrium total output increases if the domestic (foreign) government does not value producers' surplus as much as its foreign (domestic) rival does.

Proof. Using equation (14) the derivatives of equilibrium total output with respect to the domestic market share, θ , and the foreign market share, $1 - \theta$, are estimated as:

$$(26) \quad \frac{\partial Q^*}{\partial \theta} = \frac{a \gamma_d \gamma_f (\gamma_f - \gamma_d) [(m+n+2)k+2]}{(k+1) \{ [\gamma_d \gamma_f (k+3) - \gamma_d(1-\theta) - \gamma_f \theta] + \gamma_d \gamma_f k(m+n) \}^2}$$

$$(27) \quad \frac{\partial Q^*}{\partial (1-\theta)} = \frac{a \gamma_d \gamma_f (\gamma_d - \gamma_f) [(m+n+2)k+2]}{(k+1) \{ [\gamma_d \gamma_f (k+3) - \gamma_d(1-\theta) - \gamma_f \theta] + \gamma_d \gamma_f k(m+n) \}^2}$$

Since in both expressions all terms are positive except for $\gamma_f - \gamma_d$ in equation (26) and $\gamma_d - \gamma_f$ in equation (27), it is apparent that for equation (26) to be positive $\gamma_f > \gamma_d$ must be valid, and for equation (27) to be positive $\gamma_d > \gamma_f$ must be true.

When the domestic market share increases, the domestic public firm increases its output to maximize social welfare through a reduced equilibrium price and the foreign public firm decreases its output to increase the equilibrium price. However when the relative weight of producers' surplus in the domestic social welfare is more than its weight in the foreign social welfare, the domestic public firm does not match the decline in the foreign public firm's output, thus the net effect will be negative. In the meantime domestic private firms are increasing their outputs and foreign domestic private firms are decreasing theirs. The net effect here is negative as well, since domestic private firms do not increase their outputs too much avoiding a fall in the equilibrium price.

This trend is altered when the domestic government values producers' surplus less than the foreign government. The domestic government maximizes its welfare

through increased consumers' surplus. It increases its public firm output to force the net change in the total of public firms' outputs to be positive. It is interesting to notice that in these interactions public firms are used strategically to encounter a rival country's intentions by protecting the component their governments appreciate most.

4. Policy Implications

The consequences of privatization or adopting policies such as an open door policy in a mixed oligopoly have been of interest to many authors. In this section we review the outcome of implementing an open door policy, the consequences of privatizing a domestic public firm, and a coordinated privatization of both public firms when governments do not have the same approach to the producers' surplus and consumers' surplus.

4.1 Open Door Policy

It has been shown that in a mixed oligopoly the entry of a private firm reduces the public firm's output (Fjell and Pal 1996), or the total of public firms' outputs, in a multinational market (Dadpay and Heywood 2006), while it increases the total output overall. The intuition behind these arguments is that in order to preserve pre-entry producers' surplus, the public firm decreases its output to increase the equilibrium price and to offset the effect of the new entry.

To analyze the effects of open door policy equations (13) and (14) are re-estimated for the case where $m = m + 1$ or $n = n + 1$ depending on the nationality of the new entry. Comparing the new equilibrium values with the old ones facilitates analyzing the effects of a new entry. The change in the sum of public firms' outputs is estimated as the difference of pre-entry outputs and post entry outputs. Equations (28) and (29) present the results:

$$(28) \quad \frac{\Delta q_o^{d*}}{\Delta m} + \frac{\Delta q_o^{f*}}{\Delta m} = \frac{a(k+1)\{(k+4)\gamma_d \gamma_f [(1-\theta)\gamma_d + \theta\gamma_f] - [(1-\theta)\gamma_d + \theta\gamma_f]^2 - 3\gamma_d^2 \gamma_f^2 (k+1)\}}{A(A + \gamma_d \gamma_f k)}$$

$$(29) \quad \frac{\Delta q_o^{d*}}{\Delta n} + \frac{\Delta q_o^{f*}}{\Delta n} = \frac{a(k+1)\{(k+4)\gamma_d \gamma_f [(1-\theta)\gamma_d + \theta\gamma_f] - [(1-\theta)\gamma_d + \theta\gamma_f]^2 - 3\gamma_d^2 \gamma_f^2 (k+1)\}}{A(A + \gamma_d \gamma_f k)}$$

Where $A = \gamma_d \gamma_f [k^2 + (m + n + 4)k + 3] - (k + 1)[(1 - \theta)\gamma_d + \theta \gamma_f]$ and is positive. These expressions can be rewritten as:

$$\begin{aligned} \frac{\Delta q_o^{d*}}{\Delta m} + \frac{\Delta q_o^{f*}}{\Delta n} &= \frac{\Delta q_o^{d*}}{\Delta n} + \frac{\Delta q_o^{f*}}{\Delta n} \\ &= \frac{-a(k+1)[(1-\theta)\gamma_d + \theta\gamma_f - 3\gamma_d\gamma_f][(1-\theta)\gamma_d + \theta\gamma_f - \gamma_d\gamma_f(k+1)]}{A(A + \gamma_d\gamma_f k)} \end{aligned}$$

For this expression to be positive it must hold:

$$3 \leq \frac{1-\theta}{\gamma_f} + \frac{\theta}{\gamma_d} \leq k+1, \quad \text{if } k \geq 2$$

In reaction to a new entry the new firm’s respective government decreases the output of its public firm to prevent the equilibrium price from falling while the other government reacts by increasing its public firm’s output to reduce the equilibrium price. The net effect is negative if the inequality does not hold. In this case the new entrant’s respective government is successful in increasing the equilibrium price to secure a higher level of social welfare through increased producers’ surplus.

Corollary 4.1.1. The change in total output of public firms is independent of the origin of entry.

Proof. From equations (27) and (28), it is apparent that:

$$\frac{\Delta q_o^{d*}}{\Delta m} + \frac{\Delta q_o^{f*}}{\Delta n} = \frac{\Delta q_o^{d*}}{\Delta n} + \frac{\Delta q_o^{f*}}{\Delta n}$$

This is an interesting observation, since it shows that no matter what nationality the new entry has, the net effect on public firms is the same.

Proposition 4.1.1. Any new entry decreases private firms’ outputs.

Proof. Since any entry into market is an increase in the number of domestic private firms, m , or the number of foreign private firms, n , the difference between the foreign and domestic private firms’ outputs with respect to m and n can be calculated using Equation (11):

$$(30) \quad \frac{\Delta q_i^{*}}{\Delta m} = \frac{\Delta q_j^{*}}{\Delta n} = \frac{\Delta q_i^{*}}{\Delta n} = \frac{\Delta q_j^{*}}{\Delta n} = -\frac{ak\gamma_d\gamma_f[\gamma_d\gamma_f(k+1) - (1-\theta)\gamma_d - \theta\gamma_f]}{A(A + \gamma_d\gamma_f k)} < 0$$

Where $A = \gamma_d\gamma_f[k^2 + (m+n+4)k + 3] - (k+1)[(1-\theta)\gamma_d + \theta\gamma_f]$.

These differences are equal to each other. Independent of the nationality of the new entry, all private firms present in the market react similarly. In the denominator of equation (30), the expression is positive, thus the sign of expression is determined by its numerator. It is negative if:

$$(31) \quad k+1 > \frac{\theta}{\gamma_d} + \frac{1-\theta}{\gamma_f}$$

Since $0 \leq \theta \leq 1$ and $k > 0$ then the inequality holds, unless $\gamma_i, i = d$ or f , is close to zero. Since governments always attach a none zero significance to producers' surplus this is not realistic and the inequality holds.

Although a private firm's output declines, the total output still increases in some cases because of the net effect. Proposition 4.1.2 summarizes the effects of a new entry on equilibrium total output.

Proposition 4.1.2. Any entry increases total output.

Proof. Using equation (14), one can derive the change in equilibrium total output caused by a new entry:

$$(32) \quad \frac{\Delta Q^*}{\Delta m} = \frac{\Delta Q^*}{\Delta n} = \frac{a \gamma_d \gamma_f k(k+1) [\gamma_d \gamma_f (k+1) - (1-\theta)\gamma_d - \theta\gamma_f]}{A(A + \gamma_d \gamma_f k)}$$

Where $A = \gamma_d \gamma_f [k^2 + (m+n+4)k + 3] - (k+1)[(1-\theta)\gamma_d + \theta\gamma_f]$

$$\gamma_d \gamma_f (k+1) - (1-\theta)\gamma_d - \theta\gamma_f > 0 \Rightarrow k+1 > \frac{\theta}{\gamma_d} + \frac{1-\theta}{\gamma_f}$$

Since $0 < \theta < 1$ and $k > 0$, then the inequality holds.

4.2 Privatization

Pal and White (2003) and Dadpay and Heywood (2006) argue that no government has any incentive to privatize its public firm unilaterally in a multinational market. Such action reduces its social welfare and increases its rival's. They argue that to evade this prisoners' dilemma governments choose a coordinated liberalization. Dadpay and Heywood (2006) use the simultaneous liberalization of aviation industry in European Union countries as an example for such a coordinated liberalization of an industry. However the present analysis demonstrates that this is not the case anymore.

If the domestic public firm is privatized unilaterally the number of domestic private firms increases from m to $m + 1$ and if the foreign public firm is privatized the number of foreign private firms increases from n to $n + 1$. The new equilibrium values are estimated by solving the model for these cases. The following equations present the equilibrium values for private firms' outputs, the foreign public firm's output and total output, where there are $m + 1$ domestic private firms, n foreign private firms and one public foreign private firm in the market:

$$(33) \quad q_{i_dpriv}^{d*} = q_{j_dpriv}^{f*} = \frac{a[\gamma_f(k+1) - (1-\theta)]}{\gamma_f [(k+1)^2 + k(m+n+2) + m] - (k+1)(1-\theta)}$$

$$(34) \quad q_{o_dpriv}^{f*} = \frac{a[(1-\theta)(m+n) - \gamma_f(k+1-n) - \theta]}{\gamma_f [(k+1)^2 + k(m+n+2) + m] - (k+1)(1-\theta)}$$

$$(35) \quad Q_{dpriv}^* = \frac{a \gamma_f [k(m+n+2) + 2]}{\gamma_f [(k+1)^2 + k(m+n+2) + m] - (k+1)(1-\theta)}$$

Simultaneous privatization increases total number of private firms in the market to $m+n+2$, who maximize their profits simultaneously. In this case the new equilibrium values are:

$$(36) \quad q_{i-spriv}^{d*} = q_{j-spriv}^{f*} = \frac{a}{k+m+n+3}$$

$$(37) \quad Q_{spriv}^* = \frac{a(m+n+2)}{k+m+n+3}$$

As the reader may notice, when unilateral privatization takes place the equilibrium values do not depend anymore on the attitude of the government whose national firm is privatized. When both countries privatize their respective public firms, the equilibrium values depend only on the cost structure of industry and the competition in the market. This reconfirms the role of public firms as regulatory instruments as originally suggested by Defraja and Delbono (1989).

Using the pre-privatization and pro-privatization welfare levels, the change in social welfare is derived as a function of the number of firms in the market, m & n , the slope of marginal cost, k , and demand as well as the relative significance of producers' surplus in social welfare function: γ_d & γ_f . Given the complicated structure of these functions, which include several polynomials of different degrees and combinations, it is not possible to identify the sign of the change in the social welfare with authority. To consider the consequences of privatization in different market structures, numerical simulations are carried out for different combinations of m and n from 1 to 5, and for $k = 1, 5$ and 10. To study the role of relative significance of producers' surplus, simulations have been run for different values of γ_d & γ_f^2 as well (Table 1 presents the results of the numerical simulation for one of these cases).

We find several cases where either government improves the social welfare by unilaterally privatizing its public firm. This finding departs from Pal and White (2003) and Dadpay and Heywood (2006). It demonstrates that prisoners' dilemma suggested by previous authors does not always exist when governments do not have the same attitude toward producers' surplus and consumers' surplus. True there are some cases where neither government has any incentive to liberalize the industry unilaterally; however this is not a general rule anymore. For example we find out that If producers' surplus is relatively more important for the domestic government than for the foreign government, $\gamma_d > \gamma_f$, then it is less likely that the domestic government would have any incentive to privatize unilaterally. However in some other cases it does have the incentive to embark on a path of unilateral liberalization.

² The numerical simulation results are available and would be provided upon request.

Table 1

The Influence of Privatizing Domestic Public Firm on Domestic Welfare
($k = 5, a = 1, \gamma_a = 1, \gamma_f = 0.1$)

M	1	2	3	4	5
1	$\bar{-}$ $.02 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$
2	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$
3	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < .85$
4	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < .94$	$\bar{-}$ $0 < \theta < .74$
5	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < 1$	$\bar{-}$ $0 < \theta < .81$	$\bar{-}$ $.0 < \theta < .65$

The same can be said about the simultaneous or coordinated privatization efforts. While some authors argue that simultaneous privatization improves both countries' welfare, we find out several cases where this is not true. In many cases the domestic social welfare declines because of a coordinated privatization. Although there are cases where the governments can evade the prisoners' dilemma by discounting the producers' surplus, however it is not a universal principle. Again the findings of this article depart from existing literature.

There are several factors contributing to such ambiguity, we would like to highlight the role of private firms. Private firms react to either side's decision to privatize by reducing their outputs. This causes the prices to increase and consumer surplus to decline. The country which has not privatized its public firm responds by increasing its public firm's output. In some cases private firms could actually increase their profits more than the loss in consumer surplus. This results in a positive gain in social welfare which would encourage their respective government to unilaterally privatize its public firm. However this is not true for all cases.

This ambiguity might explain why several developing countries hang on to their public firms. The governments in charge might not be sure about the consequences. It is also possible that they have the strategic incentives to deny their rivals the opportunity to privatize unilaterally by discriminating between producers and consumers. This study finds out that neither the prisoners' dilemma nor the positive gains of a coordinated privatization exist in a multinational market where governments differ in their approaches to the producers' surplus. Extending the existing multinational mixed oligopoly models to encompass governments' differentiated attitudes toward producers' surplus shows that previous findings were not robust.

5. Discussion

This paper is not the first article in mixed oligopoly literature to depart from the findings of its predecessors. Many economists have utilized mixed oligopoly analysis in their studies and have produced different and sometimes contradictory results. For example Defraja and Delbono (1989) argue that in a domestic market privatization reduces social welfare unless the private sector includes several private firms. This is not the case when the market is open to foreign competition, where the nationality of private firms becomes important. Fjell and Heywood (2002) show that privatizing the public firm lowers welfare in a Stackelberg mixed oligopoly open to foreign competition if there are *fewer* domestic private firms than foreign ones, even when there are *several* private firms present in the market.

Studying multinational markets Pal and White (2003), using an intra-industry trade model, and Dadpay and Heywood (2006) demonstrate that privatizing one country's public firm reduces its social welfare but increases its rival's. However when we move away from a symmetric market structure, as demonstrated by the present study, the suggested prisoners' dilemma disappears. It seems mixed oligopoly analysis has yet to produce a robust finding regarding the consequences of privatization and its effects on social welfare. The lack of robustness in mixed oligopoly analysis dealing with privatization effects might be caused in part by assuming homogeneity in the market and in part by assuming a common quadratic cost structure for different firms. In the existing mixed oligopoly literature the foreign and domestic firms also are similar in technology and motives, facing identical demand functions.

There are several directions for potential improvements. Potential extensions would include a generalized approach to cost structure, as used by Myles (2002), or to differentiate demand for the domestic products from demand for the foreign ones, replacing perfect substitutability with a partial one. Another step would be constructing an approach to include technological differences among countries. One also should not forget about the importance of transaction costs and hidden trade tariffs in multinational markets. The last but not the least is to consider other cases for competition. Presently most authors rely on Cournot approach; firms compete and interact by determining their outputs. It seems the next natural extension for mixed oligopoly analysis is to utilize Bertrand approach, where firms interact with each other through price setting. This approach would facilitate modeling private firms' different pricing strategies and the effects of governments' preferential treatment of their citizens and private firms.

It must be noted that price already has an important role in the mixed oligopoly analysis. Reacting to increased overseas competition governments decrease their public firms' outputs to increase the equilibrium price. This is also an effort to maximize their respective societies' welfare through protecting their producers' surplus. The question still remains if a Bertrand analysis would reach similar con-

clusions or depart drastically from existing literature. Hopefully further studies will provide a robust answer to this question.

6. Conclusion

This paper was motivated by the role of governments' attitude toward producers' surplus in a multinational mixed oligopoly market. It shows that when the governments appreciate producers' surplus and consider it to be more significant than consumers' surplus, their respective public firms reduce their outputs in response to any overseas development that might lower the price. It also shows that a government's decision to privatize single handedly or to join a coordinated privatization effort depends on many factors. This decision varies from market structure to market structure.

This paper also demonstrates that public firms decrease their outputs when a country's share of consumer surplus and its respective government attaches a higher significance to producers' surplus than its rival governments do. Like all other mixed oligopoly cases, the public firm responds to entry of new rival private firms by reducing its output in order to prevent the equilibrium price from falling, however the net effect depends on the market structure and the weight of producers' surplus in the respective social welfare function. Finally, this paper shows that governments might find discriminating between producers and consumers to have a strategic value like that of subsidization in their dealings with their rivals in a mixed oligopoly. Thus governments might have some strategic motives in following a different path from that of their rivals.

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A. Appendix

A.1. All Products are Consumed by Foreign Population, $\theta = 0$

Domestic private firm output equals foreign private firm output:

$$(A1) \quad q_i^{d*} = q_i^{f*} = \frac{a[\gamma_f(k+1) - 1]}{\gamma_f[(k+2)^2 + k(m+n) - 1] - (k+1)}$$

Domestic public firm output:

$$(A2) \quad q_o^{d*} = \frac{a(k+1-m)(\gamma_f(k+1) - 1)}{(k+1)^2[\gamma_f(k+3) - 1] + \gamma_f k(k+1)(m+n)}$$

Foreign public firm output is:

$$(A3) \quad q_o^{f*} = \frac{a\{\gamma_f(k+1-n)(k+1) + k(m+n+1) + n+1\}}{(k+1)^2[\gamma_f(k+3) - 1] + \gamma_f k(k+1)(m+n)}$$

Total output is:

$$(A4) \quad Q^* = \frac{a\gamma_f[(m+n+2)k+2]}{(k+1)[\gamma_f(k+3) - 1] + \gamma_f k(m+n)}$$

Equilibrium price is:

$$(A5) \quad P^* = \frac{a(k+1)[\gamma_f(k+1) - 1]}{(k+1)[\gamma_f(k+3) - 1] + \gamma_f k(m+n)}$$

A.2. All Products are Consumed by Domestic Customers, $\theta = 1$

Domestic private firm output equals foreign private firm output:

$$(A6) \quad q_i^{d*} = q_i^{f*} = \frac{a[\gamma_d(k+1) - 1]}{\gamma_d[(k+2)^2 + k(m+n) - 1] - (k+1)}$$

Domestic public firm output:

$$(A7) \quad q_o^{d*} = \frac{a\{(k+1-m)[\gamma_d(k+1)] + [k(m+n+1) + m+1]\}}{(k+1)^2[\gamma_d(k+3) - 1] + \gamma_d k(k+1)(m+n)}$$

Foreign public firm output is:

$$(A8) \quad q_o^{f*} = \frac{a\{(k+1-n)[\gamma_d(k+1) - 1]\}}{(k+1)^2[\gamma_d(k+3) - 1] + \gamma_d k(k+1)(m+n)}$$

Total output is:

$$(A9) \quad Q^* = \frac{a\gamma_d[(m+n+2)k+2]}{(k+1)[\gamma_d(k+3) - 1] + \gamma_d k(m+n)}$$

Equilibrium price is:

$$(A10) \quad P^* = \frac{a(k+1)[\gamma_d(k+1) - 1]}{(k+1)[\gamma_d(k+3) - 1] + \gamma_d k(m+n)}$$