

MIXED OLIGOPOLY IN A SINGLE INTERNATIONAL MARKET*

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This paper departs from previous literature by considering a mixed oligopoly with two countries each with public and private firms competing in a single market. This differs from the traditional framework of examining a single domestic market in which foreign and domestic firms compete and is motivated, in part, by international airline markets but serves to characterise many markets. The resulting equilibrium emphasises that the strategic interaction of the two public firms usually serves to reduce welfare. Thus, the usual reason to imagine a public firm in a mixed oligopoly, to enhance welfare, is lost when such firms compete in the interest of their respective countries.

I. INTRODUCTION

At least since Merrill and Schneider (1966), economists have considered public firms a potential mechanism to control oligopolistic behaviour. The potential arises from the interaction of a welfare maximising public firm with profit maximising domestic private firms. Among others, see Harris and Wiens (1980), Sertel (1988), Cremer, Marchand and Thisse (1989, 1991), DeFraja and Delbono (1989, 1990), Fershtman (1990) and Nilssen and Sorgard (2002). More recently, the modelling of this mixed oligopoly has included foreign private firms as well as domestic firms. See, for example, Fjell and Pal (1996), Pal and White (1998), Fjell and Heywood (2002) and Sefahvand (2004). This inclusion adds to the model's relevance providing a structure with which to examine open-door policies, cross-border acquisition and the consequences of import tariffs and domestic subsidies. Yet, typical models examine only a single domestic country in which all consumption takes place. This limits their applicability as it excludes the possibility of domestic firms earning profit by selling to foreign customers. Similarly, it excludes the possibility of a foreign public firm selling to domestic customers in the welfare interest of a foreign country.

In this paper, we model an international mixed oligopoly in which private firms from two countries compete for customers that originate from different countries but form a single market. Public firms from each country maximise only their own country's welfare interacting with each other and with both the foreign and domestic private firms in the international competition.

To motivate this modelling consider the airline passenger markets between major international city pairs. These markets often include service by the respective national (public) firms and, increasingly, private airlines from each country. Despite recent well-publicised examples of privatisation, Doganis (2001) identifies 85 airlines retaining a state owned controlling interest as of January 2000. Traditionally bilateral treaties restrict the nationality of the airlines serving a city pair to those of the two countries involved.¹ SAS, the once fully public and now partially public airline of Scandinavia, competed on international routes with as many as three private

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¹ The major exception to this bilateralism has been the movement of the European Union toward multilateral liberalisation within the European Common Aviation Area. Yet, this movement has generally not altered bilateral relationships between European countries and countries outside of Europe and, as Doganis (2001 p. 23) points out, bilateral agreements remain the common organising principle of international aviation.

airlines from Scandinavia. These routes often included both public and private airlines from other countries. Thus, in flying to Madrid the Scandinavian airlines competed against both Iberia, the public airline of Spain, as well as against Spanair, a private Spanish airline.²

While our specific model will consider two countries, much like the airline example, our modelling applies more generally to circumstances in which firms from separate nations compete over both domestic and foreign customers in a single international market. In such 'common market' circumstances, welfare maximising public firms care about only the consumer surplus of their own citizens and the profits of their own firms. Yet, they value this consumer surplus regardless of whether it originates from a domestic or foreign firm (indeed, regardless of whether it originates from buying the product at home or abroad). Similarly, they value the profits from their domestic firms regardless of whether they originate from selling to domestic citizens or to foreigners and regardless of where those sales take place.

Specifically, our focus differs from two previous attempts that examine international trade in a mixed oligopoly. Unlike Pal and White (2003), we do not focus on the roles of tariffs and subsidies and examine, instead, a range of alternative policy issues including open door policies, cross-border acquisition and privatisation. Our model itself differs in that it allows asymmetry in the size of the two countries. We consider a single international market and by allowing for very different numbers of customers and firms within the countries, traditional insights can be reversed. Thus, we show that the foreign sales of domestic firms may be large enough that the domestic public firm will actually increase profit by reducing output and raising the price. As an illustration, a welfare maximising public airline from a country that is largely a tourist destination will recognise that its own citizens are a small share of the market and so concentrate on earning profit from the route.

More generally, we seek to capture the realistic rivalry between national firms and with it the rivalry between countries. Unlike Barcena-Ruiz and Garzon (2005), we do so in a context that does not assume that public firms are necessarily less efficient than private firms. Thus, in addition to allowing for asymmetry between countries, we remove one of the obvious reasons for privatisation. Nonetheless, the simultaneous privatisation of both domestic and foreign public firms generally increases country specific economic welfare while the privatisation of only one country's public firm typically hurts that country. Thus, much in the flavour of strategic trade models, public firms in our model play the role of trade barriers (even when not subsidised by governments) and their simultaneous elimination increases welfare.

Section II describes our model and presents the equilibrium results. Section III presents propositions that capture basic aspects of the equilibrium. Section IV uses our model to return to the policy issues that interested previous investigators and Section V presents conclusions.

II. THE MODEL AND ITS SOLUTION

Consider a market in which customers from two nations purchase a homogenous commodity. The market consists of one domestic public firm, one foreign public firm, m domestic private firms and n foreign private firms. The cost function of any firm is $C(q) = f + (1/2)kq^2$. Thus, following previous mixed oligopoly models, there is a fixed cost f and an increasing marginal cost kq , where $k > 0$ is a constant.³

² Another current example is the nonstop route from Lisbon to Madrid on which each country has a public airline, Iberia and Air Portugal, and each country has a private airline, Spanair and Portugalia Airline.

³ If costs are decreasing or constant, a public firm can simply produce the entire amount that optimises its objective function and deliver first best efficiency. Moreover, we note that Borenstein and Rose (1994) estimate that airline cost structures have increasing marginal cost in passenger miles.

Let q_o^d be the output of the public domestic firm, q_o^f be the output of the foreign public firm, q_i^d be the output of a private domestic firm ($i = 1, \dots, m$) and q_j^f be the output of a foreign private firm ($j = 1, \dots, n$). Again, following previous models we assume a linear inverse demand curve:

$$P = a - (q_o^d + q_o^f + \sum_{i=1}^m q_i^d + \sum_{j=1}^n q_j^f) \tag{1}$$

As a consequence, total consumer surplus from the market is:

$$CS = (1/2)(q_o^d + q_o^f + \sum_{i=1}^m q_i^d + \sum_{j=1}^n q_j^f)^2 \tag{2}$$

Each private firm maximises its profit, $\pi_i^d = Pq_i^d - f - (1/2)k(q_i^d)^2$ for the domestic firms and $\pi_j^f = Pq_j^f - f - (1/2)k(q_j^f)^2$ for the foreign firms. The public firms maximise their country specific welfare, the sum of their own country's consumer surplus and the total profit earned by firms from their own country:

$$W_o^d = \theta(1/2)(q_o^f + q_o^f + \sum_{i=1}^m q_i^f + \sum_{j=1}^n q_j^f)^2 + \pi_o^d + \sum_{i=1}^m \pi_i^d \tag{3}$$

$$W_o^f = (1 - \theta)(1/2)(q_o^d + q_o^f + \sum_{i=1}^m q_i^d + \sum_{j=1}^n q_j^f)^2 + \pi_o^f + \sum_{j=1}^n \pi_j^f \tag{4}$$

where $\pi_o^d = Pq_o^d - f - (1/2)k(q_o^d)^2$ and $\pi_o^f = Pq_o^f - f - (1/2)k(q_o^f)^2$ and θ is the share of total output consumed by customers from the domestic country.⁴

Jointly solving the series of first order conditions under the assumption of simultaneous quantity setting yields the following equilibrium values:

Domestic private firm output equals foreign private firm output:

$$q_i^{*d} = q_j^{*f} = \frac{ak}{k^2 + nk + mk + 3k + 2} \tag{5}$$

Domestic public firm output:

$$q_o^{*d} = \frac{a(k^2 + (1 + \theta n + \theta m + 2\theta - m)k + 2\theta)}{k^3 + (k^2 + k)(m + n) + 4k^2 + 5k + 2} \tag{6}$$

And foreign public firm output is:

$$q_o^{*f} = \frac{a(k^2 + (m + 3 - \theta m - \theta n - 2\theta)k - 2\theta + 2)}{k^3 + (k^2 + k)(m + n) + 4k^2 + 5k + 2} \tag{7}$$

Total output is:

$$Q = \frac{a[(m + n + 2)k + 2]}{k^2 + (m + n + 3)k + 2} \tag{8}$$

Equilibrium price is:

$$P^* = \frac{ka(k + 1)}{k^2 + (m + n + 3)k + 2} \tag{9}$$

We now briefly turn to evaluating the characteristics of this equilibrium.

⁴ A slightly more general formulation could allow a portion of the foreign firms' profit to be reinvested in the domestic country and a portion of the domestic firms' profit to be reinvested in the foreign country. One might also imagine that there are customers from outside the two countries so that the sum each country's share of consumer surplus need not be one.

III. BASIC RESULTS AND IMPLICATIONS

The critical new aspects of our model are two countries each with public and private firms and the parameter θ that divides consumption between the two countries. We present a series of straightforward propositions that help isolate the importance of these innovations by examining the consequences of changes in the division of customers between the two countries on public firm outputs and on country specific and global welfare.

Following from the introduction, a country with a small share of consumers will have a public firm that puts less emphasis on consumer surplus and so will be interested in producing less and with a higher price.

Proposition 3.1. $\partial q_o^d / \partial \theta > 0$ and $\partial q_o^f / \partial \theta < 0$.

Proof: From equation (6), $\frac{\partial q_o^d}{\partial \theta} = \frac{a((m+n+2)k+2)}{k^3+(k^2+k)(m+n)+4k^2+5k+2} > 0$ and from equation (7),

$$\frac{\partial q_o^f}{\partial \theta} = -\frac{a[(m+n+2)k+2]}{k^3+(k^2+k)(m+n+4)+k+2} < 0$$

Thus, the output of the domestic public firm increases as the domestic country's share of the market increases and it reaches a maximum when θ equals one (the case typically examined in mixed oligopoly models to date). As mentioned, in our airline example, if the domestic country is a tourist destination, the vast majority of customers will be those of the foreign country and θ will be small. The domestic public airline produces a relatively small quantity in order to help increase profit. The popular tourist destination for Iranians of Dubai may be an example with 42 flights a week from Tehran on private airlines but with the public Emirates Airline providing only two flights a week.

Indeed, if θ is small enough relative to the domestic country's share of firms and so of profit, the public firm will reduce output below that of a profit maximising private domestic firm.

Proposition 3.2. $q_o^d \cong q_i^d$ as $\theta \cong \frac{mk}{(m+n+2)k+2}$ and $q_o^f \cong q_i^f$ as $\theta \cong \frac{m(k+2)+2}{(m+n+2)k+2}$

Proof: $\Delta q^d = q_o^d - q_i^d = \frac{a(k^2+(1+\theta n+2\theta+\theta m-m)k+2\theta)}{k^3+(k^2+k)(m+n)+4k^2+5k+2} - \frac{ak}{k^2+nk+mk+3k+2}$ and

$$\Delta q^f = q_o^f - q_i^f = \frac{-a[(mk+nk+2k+2)\theta - mk - 2k - 2]}{[k^2+(m+n+3)k+2](k+1)}$$

setting each difference equal to zero and solving each for θ completes the proof.

Examining the right hand side of the inequality shows that as the relative size of m increases relative to n , the critical level of θ increases. Thus, the public firm of the domestic country may produce less than a private firm even with a relatively large country share. This happens when domestic firms comprise the vast majority of the market. As an illustration, given $n = 1$, $m = 6$ and $k = 1$, the critical value is 0.55 indicating for country shares below this, the domestic public firm continues to produce less than a private firm. Such an illustration goes to the heart of the supposed reason for a public firm. The original idea was that replacing a private firm with a public firm that valued consumer surplus would increase output (and welfare). Yet, when there

are sufficient foreign consumers, replacing a private firm with a public firm reduces both output and consumer surplus.

We now examine the county specific welfare consequences.

Proposition 3.3. *There exists an θ^{d*} such that if $\theta < \theta^{d*}$ then $\partial W_o^d / \partial \theta > 0$, if $\theta > \theta^{d*}$ then $\partial W_o^d / \partial \theta < 0$ and if $\theta = \theta^{d*}$ then $\partial W_o^d / \partial \theta = 0$. There also exists an θ^{f*} such that if $\theta < \theta^{f*}$ then $\partial W_o^f / \partial \theta < 0$, if $\theta > \theta^{f*}$ then $\partial W_o^f / \partial \theta > 0$ and if $\theta = \theta^{f*}$ then $\partial W_o^f / \partial \theta = 0$. And, $\theta^{d*} > \theta^{f*}$.*

Proof: Returning the equilibrium values from equations (5)–(9) to equation (3)

$$\frac{\partial W_o^{*d}}{\partial \theta} = 0 \Rightarrow \theta^{*d} = \frac{mk[(k + 1)^2 + 3] + nk(k + 1)^2 + 2k(k + 2)^2 + 2}{2k[k(m + n + 2) + 2]}.$$

It can be checked that $\partial^2 W_o^{*d} / \partial \theta^2 < 0$. An analogous condition holds for the foreign firm. Moreover, $\partial W_o^d / \partial \theta > 0$ evaluated at θ^{f*} .

Highlighting the rivalry between the countries, this proposition shows there is no single θ that maximises both domestic and foreign welfare. Each country does better with a larger share of the market than that which maximises its rival’s welfare.

IV. EXPLORING POLICY ISSUES

In this section, we consider three policy issues. First we consider the implementation of an open door policy in which the market is open to new entry. Second, we consider the implications of a cross-border acquisition. Third, we consider the policy of privatising one of the two public firms.

a) An open door policy

We consider the effects of an open door policy that allows entry of additional private firms into the mixed oligopoly. Again, an illustration comes from the airline industry. Air India competes with domestic private airlines and the Indian government explicitly considered allowing foreign private firms to enter. See Fjell and Pal (1996). Fjell and Pal (1996) use the traditional model to show that while entry of domestic firms causes the domestic public firm to reduce output, entry of foreign firms unambiguously causes the domestic public firm to increase output. This result is interesting as the addition of a firm in a Cournot model is not usually associated with other firms increasing output. Yet, the finding must be modified in our framework that allows competing public firms.

Proposition 4.1. *Entry of a domestic private firm causes output of the domestic public firm to decrease. Similarly, entry of a foreign private firm causes output of the foreign public firm to decrease. If θ and m are both small (large), entry of a foreign private firm causes output of the domestic public firm to decrease (increase). If θ is large (small) and n is small (large), entry of a domestic private firm causes output of the foreign public firm to decrease (increase).*

Proof: From equations (6) and (7), $\partial q_o^d / \partial m < 0$ and $\partial q_o^f / \partial n < 0$ but $\partial q_o^d / \partial n \cong 0$ as $m + (1 + k)(\theta - 1) \cong 0$ and $\partial q_o^f / \partial m \cong 0$ as $n - \theta(1 + k) \cong 0$.

Only when a particular country has a large share of the consumers or many private firms will the public firm in that country respond to entry by private firms from the other country by increasing its output. At an extreme, when θ is one (zero) output of the domestic (foreign) public firm will always increase in response to entry by a foreign (domestic) private firm. This follows because the public firm interacts with the private firms from its country. Thus, when a private firm from the other country enters, the private firms from its own country reduce output. The public firm responds to this reduction by increasing output.

Despite these extremes, the total output of the public firms together and the total output of all firms follow the logic expected from other Cournot models.

Corollary 4.1. $\frac{\partial q_o^{*d}}{\partial m} + \frac{\partial q_o^{*f}}{\partial m} < 0, \frac{\partial q_o^{*d}}{\partial n} + \frac{\partial q_o^{*f}}{\partial n} < 0$ and $\frac{\partial Q}{\partial m} > 0, \frac{\partial Q}{\partial n} > 0$

Proof: $\frac{dq_o^d}{dn} + \frac{dq_o^d}{dn} = \frac{dq_o^d}{dm} + \frac{dq_o^d}{dm} = -2 \frac{ak(k+1)}{(k^2 + nk + mk + 3k + 2)^2} < 0$ and $\frac{dQ}{dn} = \frac{dQ}{dm} = \frac{ak(1+k)^2}{(k^2 + nk + mk + 3k + 2)^2} > 0$

The intuition flows from the recognition that total output of all firms, Q , and so the sum of outputs from the two public firms is independent from θ (see equations (5) and (8)).⁵ The point is that if each country has a public firm, increases in entry are associated with overall reductions in output by the public firms but an increase in total output much as expected in the case of a single public firm in a single closed market (Defraja and Delbono, 1989).

Global welfare results follow:

Proposition 4.2. *Open door entry of either foreign or domestic firms generally increases global welfare.*

Proof: Add equations (3) and (4) to generate global welfare and substitute equilibrium values from equations (5)–(9) and evaluate the sign of $\partial W_o^{*g} / \partial m$ and $\partial W_o^{*g} / \partial n$. The derivatives are presented in (A1) of the Appendix.

While these derivatives cannot unambiguously be signed, simulation makes the pattern clear. By setting the above derivatives equal to zero and solving for the roots one can determine the range of θ over which entry increases global welfare. Table I presents the case for $a = 1$ and $k = 1$ showing that only for relatively large numbers of both foreign and domestic firms and only for extreme values of θ does the entry of a domestic firm cause a welfare decline. (Note that in the case of foreign entry the critical values of θ shown in the table are simply symmetric compliments, one minus θ .) Moreover for values of $k > 2.75$, all cases in Table I are unambiguously positive. Further, as k approaches zero, all values are unambiguously positive.

⁵ Alternatively, from equations (6) and (7):

$$q_o = q_o^d + q_o^f = \frac{a(k^2 + (1 + \theta n + 2\theta + \theta m - m)k + 2\theta)}{k^3 + (k^2 + k)(m + n) + 4k^2 + 5k + 2} + \frac{a(k^2 + (m + 3 - \theta m - \theta n - 2\theta)k - 2\theta + 2)}{k^3 + (k^2 + k)(m + n) + 4k^2 + 5k + 2} = \frac{2a(k + 1)}{k^2 + (m + n + 3)k + 2}$$

Table I The influence of open door entry on global welfare: when $k = 1$ and $a = 1$
 $\partial W_o^{g*} / \partial m$

M						
N	1	2	3	4	5	6
1	+	+	+	+	+	+
2	+	+	+	+	+	+ for $\theta > 0.04$
3	+	+	+	+	+ for $\theta > 0.04$	+ for $\theta > 0.08$
4	+	+	+	+ for $\theta > 0.02$	+ for $\theta > 0.07$	+ for $\theta > 0.11$
5	+	+	+	+ for $\theta > 0.04$	+ for $\theta > 0.09$	+ for $\theta > 0.13$
6	+	+	+	+ for $\theta > 0.06$	+ for $\theta > 0.10$	+ for $\theta > 0.14$

$\partial W_o^{g*} / \partial n$

M						
N	1	2	3	4	5	6
1	+	+	+	+	+	+
2	+	+	+	+	+	+
3	+	+	+	+	+	+
4	+	+	+	+ for $\theta < 0.98$	+ for $\theta < 0.96$	+ for $\theta < 0.94$
5	+	+	+ for $\theta < 0.96$	+ for $\theta < 0.93$	+ for $\theta < 0.91$	+ for $\theta < 0.90$
6	+	+ for $\theta < 0.96$	+ for $\theta < 0.92$	+ for $\theta < 0.89$	+ for $\theta < 0.87$	+ for $\theta < 0.86$

Note: + The derivative is positive over all values of θ when no range is specified.

Finally, in all cases between zero and 2.75, the vast majority of the range of θ is associated with a positive derivative. There remains only a small set of cases, as shown in Table I, when the value of θ is very large that foreign entry lowers welfare and when θ is very small that domestic entry lowers welfare.

b) Cross border acquisition

As Chakrabarty and Heywood (2004) emphasise, local equity requirements requiring majority local ownership remain in at least a dozen countries but the last two decades have seen such requirements relaxed or dropped in many countries (Thailand, India and Ghana to name but a few). Such policy changes allow increased cross-border acquisition.

In an oligopoly without public firms, such an acquisition has no influence. If there are m domestic firms and n foreign firms, and one of the domestic firms is acquired by foreign nationals, there become $n + 1$ foreign firms and $m - 1$ domestic firms but there still remain $m + n$ private firms with identical objective functions. Fjell and Pal (1996) show that this changes with the presence of a public firm in the domestic country. The acquisition lessens the importance of domestic profit in the objective function of the public firm that responds by increasing output. The consequence is an increase in consumer surplus, a decrease in profits and, on net, a decrease in welfare. Little of this logic remains when both countries have public firms. Examining equations (5) and (9) shows that if $m + n$ remains constant, neither the output of the private firms nor the total output of all firms change. Thus, the influence of foreign acquisition is simply offsetting quantity responses by the two public firms. From proposition 4.1 we know the output of the domestic public firm will increase and, as a consequence, the output of the foreign public firm decreases by an identical amount.

Proposition 4.3. *If a domestic firm is acquired by foreign nationals, there is no change in total output, the profit earned by private firms or consumer surplus. The output of the domestic public firm increases and that of the foreign public firm decreases. Total profit earned by the two public firms determines the welfare consequences of the acquisition.*

Proof: From (8) it is apparent that total output, and so price and consumer surplus, is constant as long as $m + n$ is constant. From (5) it is apparent that private firm output, and so the sum of private firm profit is constant as long as $m + n$ is constant. Thus, the welfare change is given by the change in the profits of the public firms.

In particular, the equilibrium values of q_o^{*d} and q_o^{*f} are recalculated with $m - 1$ domestic firms and $n + 1$ foreign firms. The sum of profits for the public firms is calculated and subtracted from the sum associated with the original equilibrium:

$$\Delta\pi_o^{*d} + \Delta\pi_o^{*f} = \frac{a^2k^2[2mk + 2 - k - 2\theta k(m + n) - 4\theta(k + 1)]}{[k^3 + (k^2 + k)(m + n) + 4k^2 + 5k + 2]^2} \quad (10)$$

The sign of this difference, and so the welfare change, depends on the sign of the numerator. Thus, a sufficient condition for the welfare change to be positive is that $\theta < \min[1/4, m/(m + n)]$. For small values of θ the output of the domestic firm will be very small as it maximises welfare by concentrating on profit rather than consumer surplus. On the other hand, the foreign public firm is concentrating on consumer surplus and its output is large. In this circumstance, the loss of a private domestic firm and the gain of a private foreign firm reduce the importance of profit for the domestic public firm while increasing the importance of profit for the foreign public firm. In a result similar to Proposition 3.4 this makes the outputs of the two public firms more nearly similar which increasing the sum of profits and so welfare.

c) Privatisation

Earlier models of mixed oligopoly have stressed the importance of the public firm in enhancing social welfare. In the case of a single country (no foreign firms), Defraja and Delbono (1989) show that except in the case of very large numbers of private firms the privatisation of the public firm reduces welfare. Pal and White (2003) consider two separate markets (home and foreign) in which firms separately make quantity decisions showing that privatisation of one country's public firm always lowers its welfare but increases the welfare of the other country. The results from our single international model are more ambiguous but do point in the same direction.

We first examine the case of privatisation of both public firms in our model of two countries. The equilibrium dissolves to that of a single market with $m + n + 2$ private firms in a Cournot oligopoly. The global welfare associated with equations (5)–(9), W_o^{*g} , can then be subtracted from the welfare associated with this standard Cournot case (A2 in appendix). While the difference cannot be unambiguously signed, the pattern is compelling.

Proposition 4.4. *In most relevant cases, privatisation of both public firms increases global welfare.*

Proof: The differences in the welfares, as discussed above and shown in the appendix are simulated for a wide variety of relevant cases. The simulation results for $(a = 1, k = 1)$ are shown in Table II and all but one entry is unambiguously positive.

Table II The influence of privatising both public firms on global welfare, $k = 1, a = 1$

M							
N		1	2	3	4	5	6
1		- for $0.39 < \theta < 0.62$	+	+	+	+	+
2		+	+	+	+	+	+
3		+	+	+	+	+	+
4		+	+	+	+	+	+
5		+	+	+	+	+	+
6		+	+	+	+	+	+

Notes: + Privatisation increases global welfare over all values of θ when no range is specified. - Privatisation decreases global welfare within the ranges specified.

Several additional results are apparent from simulation. First, as n and m increase for given a and k , the influence of privatisation on welfare is more likely to be unambiguously positive. Only when k is very large, and so the optimal number of firms in the industry would be extremely large, $k = 5.88$, do the majority of entries in Table II become ambiguous. Even when entries are ambiguous, the range of θ in which privatisation can increase welfare remains extremely large. When $k = 4$, 25 of 36 entries are unambiguously positive and of the remaining 11 cells, the average range of θ for which the results are positive is approximately 0.88 of potential range of 1.0.

We now consider the influence of privatisation of a single public firm, arbitrarily choosing the foreign firm, and recognising that when the foreign public firm becomes private our model moves closer to that Fjell and Pal. It becomes a case of one domestic public firm, m domestic private firms and $n + 1$ foreign private firms.⁶ Again, the global welfare associated with equations (5)–(9), W_o^{*g} , is subtracted from the welfare associated with the privatised foreign firm equilibrium (A3 in the Appendix). The simulations of this difference in Table III show that the influence is ambiguous but identifies the set of values for which privatisation causes a decline in global welfare.

Finally, we examine the influence of privatising the foreign firm on the foreign country’s welfare. Here the welfare of the foreign country when its public firm is privatised (A4 in the Appendix) is subtracted from W_f^* as associated with equations (5)–(9). As shown in Table IV, privatisation generally decreases the foreign country’s welfare and this effect is more likely when the total number of firms is smaller and country shares are more nearly equal. Analogously, if only the domestic firm were privatised, the welfare of the domestic country would, in general, decrease. Thus, we identify a serious potential free-rider problem summarised as the final proposition.

Proposition 4.5. *Typically, neither country has an incentive to unilaterally privatise its public firm.*

This finding is important as it suggests that the international competition between public firms is wasteful but that neither country has an incentive to unilaterally privatise its public firm. This may help explain the persistence of many public airlines and why Europe has been able to privatise many of their airlines only during a period of coordinated liberalisation (see fn. 1). It may also help explain why countries such as Mexico that unilaterally privatised their national airlines found themselves again nationalising those airlines.

⁶ We retain the difference that because two countries are involved there remains an allocation of customers across countries as given by θ .

Table III The influence of privatising the foreign firm on global welfare, $k = 1, a = 1$

M						
N	1	2	3	4	5	6
1	– for $0.46 < \theta < 0.83$	– for $0.71 < \theta < 0.86$	– for $0.89 < \theta < 1$	– for $0.89 < \theta < 1$	– for $0.90 < \theta < 1$	– for $0.91 < \theta < 1$
2	– for $0.47 < \theta < 0.71$	– for $0.65 < \theta < 0.75$	– for $0.78 < \theta < 1$	– for $0.83 < \theta < 1$	– for $0.82 < \theta < 1$	– for $0.83 < \theta < 1$
3	– for $0.45 < \theta < 0.62$	– for $0.59 < \theta < 0.67$	– for $0.7 < \theta < 0.75$	– for $0.73 < \theta < 1$	– for $0.75 < \theta < 1$	– for $0.77 < \theta < 1$
4	– for $0.42 < \theta < 0.56$	– for $0.54 < \theta < 0.60$	– for $0.63 < \theta < 0.66$	– for $0.67 < \theta < 0.81$	– for $0.69 < \theta < 1$	– for $0.71 < \theta < 1$
5	– for $0.39 < \theta < 0.5$	– for $0.49 < \theta < 0.55$	– for $0.58 < \theta < 0.59$	– for $0.61 < \theta < 0.70$	– for $0.64 < \theta < 0.83$	– for $0.67 < \theta < 1$
6	– for $0.36 < \theta < 0.45$	– for $0.46 < \theta < 0.50$	– for $0.53 < \theta < 0.54$	– for $0.57 < \theta < 0.63$	– for $0.6 < \theta < 0.72$	– for $0.36 < \theta < 0.83$

Note: – Privatisation decreases global welfare within the ranges specified.

Table IV The influence of privatising the foreign firm on foreign welfare, $k = 1, a = 1$

M						
N	1	2	3	4	5	6
1	– for $0 < \theta < .83$	– for $0 < \theta < .86$	– for $0 < \theta < 0.87$	– for $0 < \theta < 0.89$	– for $0 < \theta < 0.9$	– for $0 < \theta < 0.91$
2	– for $0 < \theta < .71$	– for $0 < \theta < .75$	– for $0 < \theta < 0.78$	– for $0.04 < \theta < 0.8$	– for $0.15 < \theta < 0.82$	– for $0.29 < \theta < 0.83$
3	– for $0.15 < \theta < 0.62$	– for $0.20 < \theta < 0.67$	– for $0.26 < \theta < 0.75$	– for $0.31 < \theta < 0.73$	– for $0.36 < \theta < 0.75$	– for $0.41 < \theta < 0.77$
4	– for $0.33 < \theta < 0.55$	– for $0.35 < \theta < 0.60$	– for $0.38 < \theta < 0.64$	– for $0.41 < \theta < 0.67$	– for $0.44 < \theta < 0.69$	– for $0.47 < \theta < 0.71$
5	– for $0.36 < \theta < 0.5$	– for $0.39 < \theta < 0.55$	– for $0.42 < \theta < 0.58$	– for $0.44 < \theta < 0.62$	– for $0.46 < \theta < 0.64$	– for $0.49 < \theta < 0.67$
6	– for $0.36 < \theta < 0.45$	– for $0.39 < \theta < 0.50$	– for $0.41 < \theta < 0.54$	– for $0.44 < \theta < 0.57$	– for $0.46 < \theta < 0.60$	– for $0.49 < \theta < 0.62$

Note: – Privatisation decreases foreign welfare within the ranges specified.

V. CONCLUSIONS

This paper was motivated by the realisation that previous mixed oligopoly models need to be extended to allow for the presence of both foreign consumers and a foreign public firm. Our results are symmetric for the foreign and domestic markets and we will briefly summarise those for the domestic market. The output of the domestic public firm increases as the domestic share of customers, θ , increases. Nonetheless, depending on how large that share is compared to the number of domestic versus foreign firms, the domestic public firm may produce less or more than a domestic private firm. There exists a national rivalry in θ as there is no one value that simultaneously maximises welfare in each country.

The entry of a domestic private firm causes the output of the domestic public firm to decrease but the influence on the output of the foreign public firm depends on the consumer and production shares of each country. Such entry generally increases welfare. Acquisition of a domestic firm by foreign nationals causes no change in total output as the increases by the domestic public firm are offset by the decreases of the foreign public firm. The welfare effect depends on the consumer and production shares of each country. Finally, countries face a prisoners' dilemma deciding about privatisation of their public firms. Privatising both public firms generally increases welfare but unilateral privatisation usually decreases that country's welfare and often increases the rival country's welfare.

APPENDIX

$$\frac{\partial W_o^{g*}}{\partial m} = \frac{a^2[2\theta^2(m+n+4) - \theta(mn+8n+6m+20) + mn+3m+n-10]}{2(m+n+6)^3} \tag{A1}$$

$$\frac{\partial W_o^{g*}}{\partial n} = \frac{a^2[2\theta^2(m+n+4) + \theta(mn+2n+4m+4) - m^2 + mn - 5m - n - 22]}{2(m+n+6)^3}$$

The equilibrium level of global welfare in a completely privatised market with $n + 1$ foreign firms and $m + 1$ domestic firms:

$$W^{*g}(\text{PRI2}) = \frac{a^2[(m+n)(m+n+k+6) + 2k+8]}{2(m+n+k+3)^2} \tag{A2}$$

The equilibrium level of global welfare in a market with $n + 1$ foreign private firms, m domestic private firms and one public domestic firm:

$$W_g^*(\text{PRI}_f) = \frac{a^2\{-\theta^2(m+n+1)(m+n+2) + \theta[(m-4)(m+2n+4) + m^2] + (n+4)(4m-n+4) + n(n+28) + 32\}}{2\{m+2(n-\theta) + 8\}^2} \tag{A3}$$

The equilibrium level of foreign welfare in a market with $n + 1$ foreign private firms, m domestic private firms and one public domestic firm:

$$W_f^*(\text{PRI}_f) = \frac{a^2\{(3\theta - \theta^2)(m+n+1)^2 + \theta[(n+1)(n+9-4\theta n-4\theta) + 4n-\theta m] + (m-4)^2 - 28\}}{2[m+2(n-\theta) + 8]^2} \tag{A4}$$

REFERENCES

- Barcena-Ruiz, J.C. and Garzon, M.B. 2005, 'International Trade and Strategic Privatization', *Review of Development Economics*, vol. 9, pp. 502–513.
- Borenstein, S. and Rose, N. 1994, 'Competition and Price Dispersion in the US Airline Industry', *Journal of Political Economy*, vol. 102, pp. 653–683.
- Chakrabarty, A. and Heywood, J.S. 2004, 'Foreign Investment, Vertical Integration and Local Equity Requirements', *Economica*, vol. 57, pp. 574–599.
- Cremer, H., Marchand, M. and Thisse, J. 1989, 'The Public Firm as an Instrument for Regulating an Oligopolistic Market', *Oxford Economic Papers*, vol. 41, pp. 283–301.
- , — and —. 1991, 'Mixed Oligopoly with Differentiated Products', *International Journal of Industrial Organization*, vol. 9, pp. 43–53.
- DeFraja, G. and Delbono, F. 1989, 'Alternative Strategies of a Public Enterprise in Oligopoly', *Oxford Economic Papers*, vol. 41, pp. 302–311.
- and —. 1990, 'Game Theoretic Models of Mixed Oligopoly', *Journal of Economic Surveys*, vol. 4, pp. 1–17.
- Doganis, R. 2001, *The Airline Business in the 21st Century*, Routledge, London.
- Fjell, K. and Heywood, J.S. 2002, 'Public Stackelberg Leadership in a Mixed Oligopoly with Foreign Firms', *Australian Economic Papers*, vol. 41, pp. 267–81.
- and Pal, D. 1996, 'Mixed Oligopoly with Foreign Private Firms', *Canadian Economic Journal*, vol. 29, pp. 737–743.
- Fershtman, C. 1990, 'The Interdependence between Ownership Status and Market Structure: The Case of Privatization', *Economica*, vol. 57, pp. 319–28.
- Harris, R.G. and Wiens, E.G. 1980, 'Government Enterprise: An Instrument for Internal Regulation of Industry', *Canadian Journal of Economics*, vol. 13, pp. 125–32.
- Merril, W.C. and Schneider, N. 1966, 'Government Firms in Oligopoly Industries: A Short Run Analysis', *Quarterly Journal of Economics*, vol. 80, pp. 400–12.
- Nilssen, T. and Sorgard, L. 2002, 'A Public Firm Challenged by Entry: Duplication or Diversity?' *Regional Science and Urban Economics*, vol. 32, pp. 259–274.
- Pal, D. and White, M. 1998, 'Mixed Oligopoly, Privatization, and Strategic Trade Policy', *Southern Economic Journal*, vol. 65, pp. 264–81.
- . 2003, 'Intra-Industry Trade and Strategic Trade Policy in the Presence of Public Firms', *International Economic Journal*, vol. 17, pp. 29–41.
- Sepahvand, M. 2004, 'Public Enterprise Strategies in a Market Open to Domestic and International Competition', *Annales D'Economie et de Statistique*, vol. 75/76, pp. 135–154.
- Sertel, M. 1998, 'Regulation by Participation,' *Journal of Economics*, vol. 48, pp. 114–34.